Journal of Engineering Physics and Thermophysics, Vol. 78, No. 5, 2005

INVESTIGATION OF CUTTING FORCES BY THE METHOD OF STATISTICAL PLANNING OF THE EXPERIMENT

A. I. Sagradyan UDC 621.902.31

With the method of mathematical planning, the character and the degree of influence of the parameters of cutting regimes of the corresponding cutting forces in machining steels and alloys differing in physicochemical properties have been investigated. The dependence of the corresponding cutting forces on the change in the cutting regimes has been obtained. The optimum force characteristics have been found experimentally as a result of choosing the most favorable cutting regimes.

The character and the degree of influence of the parameters of cutting regimes on the cutting-force components in machining steels and alloys differing in their physicochemical properties (steel 45, 2Kh13, R18, VT5) with cutting tools from speed-cutting alloys V14M7K25 have been investigated using the method of mathematical planning of the experiment [1–4], which has made it possible to obtain the dependence of the corresponding forces on the change in the cutting regimes. On the basis of the experiments, the power dependences of the cutting-force components on the feed rate and depth in a wide range of change in the cutting regimes have been obtained.

The values of the cutting-force components obtained by such a method in such a wide range of cutting regimes can be used both for comparative evaluation, in particular, in machining one steel with different tool materials, and for various materials with cutting tools from the speed-cutting alloy V14M7K25.

Not only does this method permit quick and fairly exact determination of the values of the cutting-force components, but it also makes it possible to achieve a marked decrease in the consumption of the material being machined and the tool material.

The experiments were performed on a 1K62 machine tool equipped with a VR-1 variable-speed drive. The output signals were obtained with the aid of a UDM-1 three-component dynamometer with subsequent recording through a TA-5 amplifier by a K-105 oscillograph.

Mathematical planning was carried out in accordance with [3, 4] for a complete factorial experiment of the type of 3^3 with splitting of the plan into three blocks with nine combinations of conditions in each. In this case, one can distinguish 13 effects with two degrees of freedom each: VSt; VS, VS^2, Vt, Vt^2, St; St^2, V^2St; VS^2t; VSt^2 and VS^2t^2, V^2S^2t, V^2St^2.

Mixing VSt^2 interactions with the three blocks, we get

$$I = VSt^2 , \quad L = X_1 + X_2 + 2X_3 . \tag{1}$$

The blocks have the following form:

$L = 0$	000	011	022	101	112	120	210	221	202
$L = 1$	100	111	122	201	212	220	010	021	002
$L = 2$	200	211	222	001	012	020	110	121	102

If only one of these blocks, e.g., $L = 1$, is realized, then the plan of the 3^3 factorial experiment third-replicate will take on the form given in Table 1.

The values of V_m, S_m, and t_m can be determined by the formulas proposed in [4]:

State Engineering University of Armenia, 105 Teriyan Str., Erevan. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 78, No. 5, pp. 190–195, September–October, 2005. Original article submitted April 20, 2005.

 1062-0125/05/7805-1034

TABLE 1. Plan of the Third Replicate of the 3^3-Type Factorial Experiment

Experiment number	Experiment code	V	S	t
1	002	V_{min}	S_{min}	t_{max}
2	010	V_{min}	S_m	t_{min}
3	021	V_{min}	S_{max}	t_m
4	100	V_m	S_{min}	t_{min}
5	111	V_m	S_m	t_m
6	122	V_m	S_{max}	t_{max}
7	201	V_{max}	S_{min}	t_m
8	212	V_{max}	S_m	t_{max}
9	220	V_{max}	S_{max}	t_{min}

TABLE 2. Plan of Experiments in Turning with V14M7K25 Cutting Tools

Experiment number	Steel 45			2Kh13			R18			VT5		
	V	S	t	V	S	t	V	S	t	V	S	t
1	20	0.12	2.50	36	0.12	2.50	5.0	0.12	2.50	10.0	0.12	2.50
2	20	0.26	0.50	36	0.26	0.50	5.0	0.25	0.50	10.0	0.25	0.50
3	20	0.61	1.12	36	0.61	1.12	5.0	0.52	1.12	10.0	0.52	1.12
4	40	0.12	0.50	60	0.12	0.50	12.2	0.12	0.50	17.3	0.12	0.50
5	40	0.26	1.12	60	0.26	1.12	12.2	0.25	1.12	17.3	0.25	1.12
6	40	0.61	2.50	60	0.61	2.50	12.2	0.62	2.50	17.3	0.52	2.50
7	80	0.12	1.12	100	0.12	1.12	30.0	0.12	1.12	30.0	0.12	1.12
8	80	0.26	2.50	100	0.26	2.50	30.0	0.25	2.50	30.0	0.25	2.50
9	80	0.61	0.50	100	0.61	0.50	30.0	0.52	0.50	30.0	0.52	0.50

$$X_1 = \frac{2\,(\ln V - \ln V_{max})}{\ln V_{max} - \ln V_{min}} + 1\,,\quad X_2 = \frac{2\,(\ln S - \ln S_{max})}{\ln S_{max} - \ln S_{min}} + 1\,,\quad X_3 = \frac{2\,(\ln t - \ln t_{max})}{\ln t_{max} - \ln t_{min}} + 1\,. \tag{2}$$

For the convenience of using matrix algebra, in the subsequent calculations the levels of cutting regimes V, S, and t have been coded as follows: to the upper level corresponds +1, to the medium level — 0, and to the lower level – 1. Thus, at

$$V = V_{max}\quad X_1 = 1\,,\quad S = S_{max}\quad X_2 = 1\,,\quad t = t_{max}\quad X_3 = 1\,;$$

$$V = V_m\quad X_1 = 0\,,\quad S = S_m\quad X_2 = 0\,,\quad t = t_m\quad X_3 = 0\,;$$

$$V = V_{min}\quad X_1 = -1\,,\quad S = S_{min}\quad X_2 = -1\,,\quad t = t_{min}\quad X_3 = -1\,.$$

The plan of the experiments for cutting tools from the V14M7K25 alloy in machining various materials is presented in Table 2. Performing experiments under such conditions, we obtain for each experiment the values of the cutting-force components (Table 3).

The dependence of the cutting-force components will be written in general form as

$$P_z = C_{P_z} V^{z_1} S^{y_1} t^{x_1}\,,\quad P_y = C_{P_y} V^{z_2} S^{y_2} t^{x_2}\,,\quad P_x = C_{P_x} V^{z_3} S^{y_3} t^{x_3}\,. \tag{3}$$

To determine the coefficients C_{P_z}, C_{P_y}, and C_{P_x}, as well as the exponents $z_{1,2,3}$, $y_{1,2,3}$, and $x_{1,2,3}$, the least-square technique was used [5].

TABLE 3. Experimental Data for Turning Various Materials with V14M7K25 Cutting Tools

Experiment number	Steel 45			2Kh13			R18			VT5		
	P_z	P_y	P_x	P_z	P_y	P_x	P_z	P_y	P_x	P_z	P_y	P_x
1	920	320	280	1290	680	450	1200	730	600	770	380	300
2	450	170	100	470	210	130	550	240	150	420	330	200
3	1640	650	370	1800	930	600	1650	1190	580	1150	560	370
4	250	100	60	320	170	140	290	120	100	180	70	450
5	870	290	220	930	560	300	820	450	350	750	410	320
6	2750	1390	780	3580	1610	570	4250	2230	1800	1930	870	740
7	420	150	120	470	230	180	630	330	300	340	180	140
8	1790	810	670	1600	850	470	2100	1080	850	1500	810	630
9	760	300	140	1100	600	330	900	550	200	680	420	220

TABLE 4. Coefficients C_{P_z}, C_{P_y}, C_{P_x} and Exponents $z_{1,2,3}$, $y_{1,2,3}$, and $x_{1,2,3}$ in Turning Steel 45, 2Kh13, R18, and VT5 with V14M7K25 Cutting Tools

Coefficients and exponents	Steel 45	2Kh13	R18	VT5
C_{P_z}	235.0	350.9	257.9	202.1
C_{P_y}	87.6	143.8	175.8	125.5
C_{P_x}	40.6	72.5	72.9	78.5
z_1	–0.038	–0.09	0.010	–0.02
z_2	0.007	–0.04	–0.010	–0.04
z_3	0.010	–0.074	0.004	–0.04
y_1	0.730	0.740	0.760	10.79
y_2	0.820	0.720	0.890	0.86
y_3	0.610	0.470	0.550	0.79
x_1	0.820	0.780	0.890	0.79
x_2	0.880	0.780	0.970	0.69
x_3	1.060	0.640	1.180	0.89

Taking the logarithm of Eq. (3) and introducing corresponding notations yield

$$P_1 = C_1 + z_1 V + y_1 S + x_1 t\ , \quad P_2 = C_2 + z_2 V + y_2 S + x_2 t\ , \quad P_3 = C_3 + z_3 V + y_3 S + x_3 t\ . \tag{4}$$

According to [3, 4], in our case, using also the Gaussian conditions and programming in processing the results of the investigations given in Table 3, we obtain the values of the corresponding coefficients and exponents (Table 4).

In turning steel 45 and various hard-to-machine materials, in particular, 2Kh13, R18, and VT5 with V14M7K25 cutting tools (Figs. 1–3, the curves have been plotted on the basis of the calculation data according to (1)–(4)), we noted that the largest value for all components is observed in turning R18 and the least one — in turning the titanium alloy VT5. The reason for such a phenomenon can be explained by the fact that in cutting R18 the shrinkage value is larger, and in turning VT5 — vice versa. It has also been revealed (Fig. 1) that while in turning steel 45, R18, and VT5 the cutting speed does not influence the value of the vertical component P_z in the investigated range, in turning 2Kh13 in the range of cutting speeds from 20 to 100 m/min, P_z decreases from 1850 to 1600 N. The influence of the cutting speed on P_y and P_x is insignificant.

Such a phenomenon is likely to be due to the absence of build-up in the process of turning the VT5 titanium alloy by cutting tools from speed-cutting alloys. In the entire range, with increasing speeds the cutting forces decrease, since in this case the influence of the cutting temperature manifests itself.

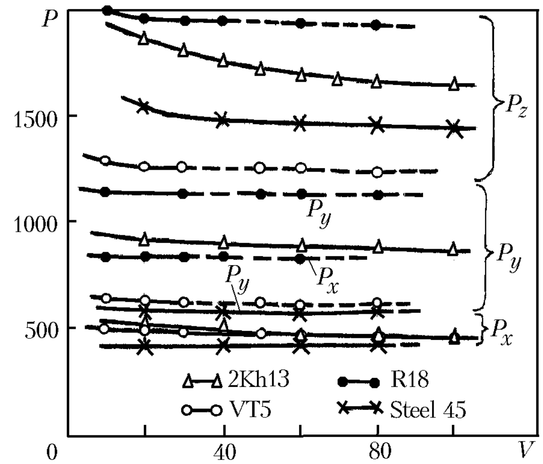
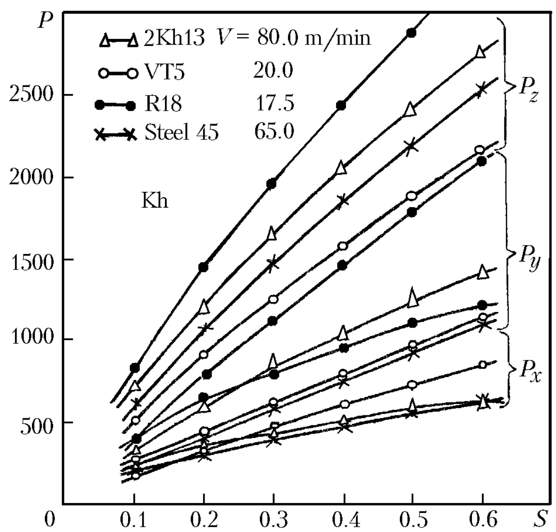


Fig. 1. Components of the cutting forces versus the cutting speed in machining different materials (S = 0.3 mm/rev, t = 2 mm). P, N; V, m/min.



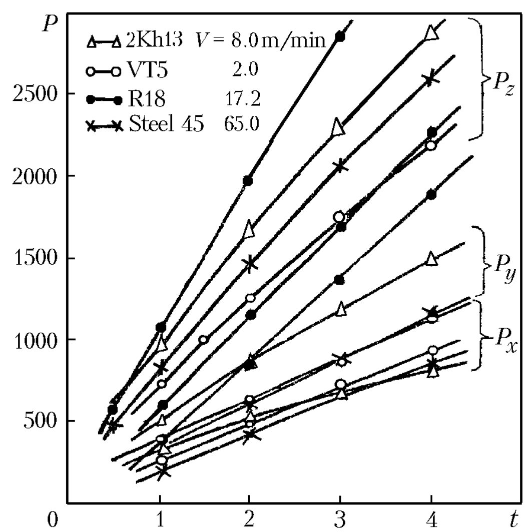


Fig. 2. Components of the cutting forces versus the feed in machining different materials (t = 2 mm). P, N; S, mm/rev.

Fig. 3. Components of the cutting forces versus depth in machining different materials (S = 0.3 mm/rev). P, N; t, mm.

Characteristically, there is no direct relation between the cutting forces and the mechanical properties of the material being machined. For example, in turning the VT5 titanium alloy in which the ultimate strength σ = 900 MPa, 2Kh13 with σ = 600 MPa, and R18 with σ = 900 MPa, the vertical components, as is seen from Fig. 1, under the same cutting conditions (V = 40 m/min, S = 0.3 mm/rev, t = 2.0 mm) have the following values: P_z = 1300, 1800, and 1980 N, respectively. This is explained by the fact that the vertical projection of the cutting force largely depends on the unit work of plastic deformation, and the latter increases with increasing deformation and stresses in the chip formation zone.

The curves in Fig. 1, as opposed to the traditional, single-factor experiment, show the absence of extremality of the P_z, P_y, and $P_x - f(V)$ curves in the investigated range of cutting speeds, and the results obtained by this method can be used to calculate the dynamic characteristics of the machine and the cutting tool.

Analyzing the dependences of the cutting forces on the feed rate and the cutting depth (Figs. 2, 3), we noted that the increase in the cutting forces with increasing rate of feed is smaller than with increasing cutting depth. This

TABLE 5. Plan of Unification over the Factor t Level

Factor S	Factor V		
	0	1	2
0	2	0	1
1	0	1	2
2	1	2	0

TABLE 6. Plan of Dispersion Analysis

Factor S	Factor V			$T_{..k}$
	0	1	2	
0	–9	–68	–52	–129
1	–44	–11	64	9
2	29	107	–18	118
$T_{i..}$	–24	28	–6	$T_{...} = -2$
$T_{.j.}$	$t = 0$	$t = 1$	$t = 2$	
	–130	–34	162	

TABLE 7. Results of the Dispersion Analysis with Full-Block Planning of the Latin Square

Source of variability	Number of degrees of freedom	Sum of squares	Mean square
Change in V	2	464.9	232.45
Change in S	2	10,214.9	5107.45
Change in t	2	14,766.2	7383.1
Error	2	1729.5	864.7
Total	8	27,175.5	

is due to the fact that with varying cutting depth the shrinkage remains practically unchanged, while with increasing or decreasing rate of feed it changes appreciably.

As is seen from Figs. 1–3, the values of the P_z components are larger for cutting R18 and 2Kh13 materials. In all events, at $V = V_{\text{m}}$, $S = S_{\max}$, and $t = t_{\max}$ (see Table 3) the most unfavorable cutting conditions are obtained. Consequently, to avoid vibrations and deformation of the piece being worked under the action of the cutting forces, this regime should be excluded.

To increase the reliability of the results of the investigations, we have tested the hypothesis on the influence of the cutting conditions on the cutting-force components by the method of dispersion analysis.

Proceeding from the investigation plan, nine experiments (Table 1) can be grouped according to the levels of the factor t (Table 5).

In testing the hypothesis, analysis is performed only for P_z in turning VT5 by cutting tools from V14M7K25 (see Table 3) (analogously for all kinds of tools and worked material, as well as for the components P_y and P_x of the cutting force). Subtracting from each reading of precoded P_z 860 N (mean value of P_z), with the F statistics remaining unaltered, we obtain the data given in Table 6.

The total sum of squares is calculated by the formula

$$SS = \sum_{j=1}^{9} \sum_{i=1}^{9} X_{ijk}^2 - \frac{T_{...}^2}{N} = 27{,}175.6 \,. \tag{5}$$

The sum of squares for different variants of tests is calculated as

$$SS_V = \sum_{i=1}^{3} \frac{T_{i..}^2}{3} - \frac{T_{...}^2}{9} = 464.9 \,, \quad SS_S = \sum_{k=1}^{3} \frac{T_{..k}^2}{3} - \frac{T_{...}^2}{9} = 10{,}214.9 \,, \quad SS_t = \sum_{j=1}^{3} \frac{T_{.j.}^2}{3} - \frac{T_{...}^2}{9} = 14{,}766.2 \,. \tag{6}$$

The sum of squares for the error is

$$SS_{er} = SS - SS_V - SS_S - SS_t = 1729.5 \,. \quad (7)$$

The results of the dispersion analysis for the given example with full-block planning of the Latin square are presented in Table 7.

Using them, one can test the following hypotheses:

$$V = 0 \quad \text{(no influence of speed)}\,, \quad F_{2'2} = 0.26\,,$$

the hypothesis is not rejected, since this quantity is minimal at a 1% significance level;

$$S = 0 \quad \text{(no influence of feed)}\,, \quad F_{2'2} = 5.9\,,$$

the quantity is substantial at a 1% significance level, the hypothesis is rejected;

$$t = 0 \quad \text{(no influence of cutting depth)}\,, \quad F_{2'2} = 8.53\,,$$

the quantity is substantial at a 1% significance level, the hypothesis is rejected.

Analyzing the foregoing, we can confirm that the cutting force in the investigated range of cutting speeds is strongly influenced by S and t, whereas the influence of the cutting speed V on the cutting-force components is insignificant (Fig. 1), which is confirmed by the calculation of the F statistics.

Using this method, one can not only quickly and fairly exactly determine the values of the cutting-force components but also achieve a marked decrease in the consumption of machined and tool materials.

For each material being machined, specific combinations of cutting regimes V, S, and t with which the machining process is not recommended have been established.

On the basis of the experiments, optimum force characteristics have been obtained for choosing the most favorable cutting regimes.

NOTATION

C_{P_z}, C_{P_y}, C_{P_x}, coefficients of cutting-force components; $F_{2'2}$, Fisher criterion; N, number of observations of the experiment; P_z, P_y, P_x, values of the vertical, radial, and axial components of the cutting force; S, feed rate, mm/rev; S_{max}, S_m, and S_{min}, maximum, mean, and minimum values of the feed rate, mm/rev; SS, total sum of squares; SS_V, SS_S, and SS_t, sum of squares for different kinds of tests for speed, feed rate, and cutting depth; T, total sum of observation differences for T_i, T_j, T_k; t, cutting depth; t_{max}, t_m, and t_{min}, maximum, mean, and minimum values of the cutting depth; V, cutting speed, m/min; V_{max}, V_m, and V_{min}, maximum, mean, and minimum values of the cutting speed, m/min; X_1, X_2, and X_3, codes of the speed, feed rate, and cutting depth, respectively; $z_{1,2,3}$, $y_{1,2,3}$, and $x_{1,2,3}$, exponents of the speed, feed rate, and cutting depth, respectively. Subscripts: m, mean; max, maximum; min, minimum; er, error.

REFERENCES

1. V. F. Bobrov, *Principles of the Theory of Metal Cutting* [in Russian], Mashinostroenie, Moscow (1985).
2. M. F. Poletika, *Contact Loadings on the Cutting Surfaces of the Tool* [in Russian], Mashinostroenie, Moscow (1985).
3. C. R. Hicks, *Fundamental Concepts of Experiment Planning* [Russian translation], Mir, Moscow (1967).
4. P. G. Katsev, *Statistical Methods for Studying the Cutting Tool* [in Russian], Mashinostroenie, Moscow (1968).
5. G. Korn and T. Korn, *Handbook of Mathematics* [Russian translation], Nauka, Moscow (1973).